Branch FE(ALL) Course:- Applied Mathematics-II

Course code : FEC201 Marks: 80 3hrs.

Q.1 A) Solve $(2x-y+1)dx+(2y-x-1)dy=0$

→ Here $M=2x-y+1$ $N=2y-x-1$

$\therefore \frac{\partial M}{\partial y}=-1$, $\frac{\partial N}{\partial x}=-1$

$\therefore \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ $\therefore$ Given D.E is exact

$\therefore$ its soln is

$\int_{y\text{-const}} M\,dx+\int(\text{Terms in } N \text{ free from } x)\,dy=c$

$\therefore \int(2x-y+1)\,dx+\int(2y-1)\,dy=c$

$\therefore x^2-xy+x+y^2-y=c$

Q.1 B) Solve $(D^2-16)y=e^{3x}+\sin 2x$

→ The A.E is $m^2-16=0$

$\therefore m^2=16$. $\therefore m=\pm 4$

$\therefore C.F=c_1e^{4x}+c_2e^{-4x}$

$P.I=\frac{1}{f(D)}x=\frac{1}{D^2-16}(e^{3x}+\sin 2x)$

$=\frac{1}{(3)^2-16}e^{3x}+\frac{1}{(-2^2)-16}\sin 2x$

$=\frac{e^{3x}}{-7}-\frac{1}{20}\sin 2x$

$\therefore G.S=C.F+P.I=c_1e^{4x}+c_2e^{-4x}-\frac{e^{3x}}{7}-\frac{\sin 2x}{20}$

Q.1 c. Evaluate $\int_0^2 x^2(2-x)^3\,dx$

$\rightarrow$ put $x = 2t$

$\therefore dx = 2\,dt$ when $x=0,\ t=0$

$x=2,\ t=1$

$$\therefore I = \int_0^1 (2t)^2(2-2t)^3 \cdot 2\,dt$$

$$= 2^{2+3+1}\int_0^1 t^2(1-t)^3\,dt$$

$$= 2^6 \cdot B(3,4)$$

$$= 64\,\frac{\Gamma 3\,\Gamma 4}{\Gamma 7}$$

$$= 64\,\frac{(2)(3!)}{6!} = \frac{\overset{16}{\cancel{64}}(\cancel{2})(\cancel{3!})}{\cancel{6}\times 5\times \overset{}{\cancel{4}}\times(\cancel{3!})}$$

$$= \frac{16}{15}$$

Q. d) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dz\,dy\,dx$

$$= \int_0^{\log 2}\int_0^{x} e^{x+y}\left(e^z\right)_0^{x+y}\,dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x} e^{x+y}\left(e^{x+y}-1\right)dy\,dx$$

$$= \int_0^{\log 2}\int_0^{x}\left(e^{2(x+y)} - e^{x+y}\right)dy\,dx$$

$$= \int_0^{\log 2} \left(\frac{e^{2(x+y)}}{2} - e^{x+y} \right)_0^x dx$$

$$= \int_0^{\log 2} \left(\frac{e^{4x}}{2} - e^{2x} - \frac{e^{2x}}{2} + e^{x} \right) dx$$

$$= \left(\frac{e^{4x}}{8} - \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + e^{x} \right)_0^{\log 2}$$

$$= \left(\frac{e^{4\log 2}}{8} - \frac{e^{2\log 2}}{2} - \frac{e^{2\log 2}}{4} + e^{\log 2} \right) - \left(\frac{1}{8} - \frac{1}{2} - \frac{1}{4} + 1 \right)$$

$$= 2 - 2 - 1 + 2 - \left(\frac{1-4-2+8}{8} \right)$$

$$= -3 + 4 - \frac{3}{8} = 1 - \frac{3}{8} = \frac{5}{8}.$$

Q.1 E. $f(x,y) = x+y$, $y(1) = 1$, $h = 0.1$

$y_0 = 1, x_0 = 1$

We know, Euler's Method

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$x_1 = x_0 + h$
$= 1.1$
$x_2 = 1.2$
$x_3 = 1.3$
$x_4 = 1.4$

put $n=0$, $y_1 = y_0 + hf(x_0, y_0)$
$= 1 + (0.1)(1+1)$
$= 1 + 0.2 = 1.2$

put $n=1$, $y_2 = y_1 + hf(x_1, y_1)$
$= 1.2 + (0.1)(x_1 + y_1)$
$= 1.2 + (0.1)(1.1 + 1.2)$
$= 1.43.$

put $n=2$, $y_3 = y_2 + hf(x_2, y_2)$
$= 1.43 + (0.1)(1.2 + 1.43).$
$= 1.6930$

Put $n=3$. $y_4 = y_3 + h \cdot f(x_3, y_3)$

$= 1.6930 + (0.1)(1.3 + 1.6930)$

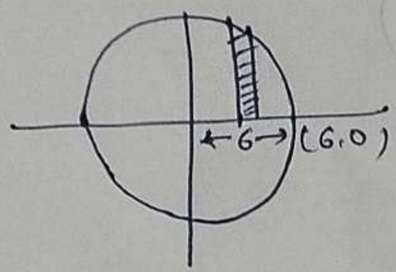
$= 2.2623$.

Put $n=4$, $y_5 = y_4 + h f(x_4, y_4)$

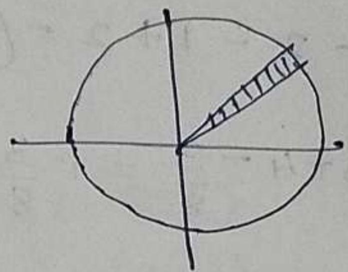
$= 2.2623 + (0.1)(1.4 + 2.2623)$

$y_5 = 2.6285 = y(1.5)$

Q.1 f) $x^2 + y^2 = 36$, a circle with centre $(0,0)$ & radius is 6



Method 1



Method 2

Method 1

y varies from $y=0$ to $\sqrt{36-x^2}$

x varies from $x=0$ to $x=6$

$\therefore$ Area $= 4 \int_0^6 \int_0^{\sqrt{36-x^2}} dy\, dx$

$= 4 \int_0^6 [y]_0^{\sqrt{36-x^2}} dx$

$= 4 \int_0^6 \sqrt{36-x^2}\, dx$

$= 4 \cdot \left[\frac{x}{2}\sqrt{36-x^2} + \frac{36}{2} \sin^{-1}\left(\frac{x}{6}\right) \right]_0^6$

$= 4 \left[18 \sin^{-1} 1 - 0 \right]$

$= 4 \times 18 \times \pi/2 = 36\pi$.

Method 2

put $x = r\cos\theta$

$y = r\sin\theta$

$dx\,dy = r\,dr\,d\theta$

$\therefore$ Area $= 4 \int_0^{\pi/2} \int_0^6 r\, dr\, d\theta$

$= 4 \int_0^{\pi/2} \left(\frac{r^2}{2}\right)_0^6 d\theta$

$= 4\left(\frac{36}{2}\right) \int_0^{\pi/2} d\theta$

$= 72\, (\theta)_0^{\pi/2}$

$= 72 \times \pi/2$

$= 36\pi$

Q.2 a) i) Solve $(x^2+1)\frac{dy}{dx} + 2xy = \frac{1}{x^2+1}$

Given Diff. Equation is Reducible to linear

Divide by x^2+1 ∴ we get

$$\frac{dy}{dx} + \frac{2x}{x^2+1} \cdot y = \frac{1}{(x^2+1)^2}$$

which is of the form $\frac{dy}{dx} + Py = Q$

$$P = \frac{2x}{x^2+1}, \quad Q = \frac{1}{(x^2+1)^2}$$

$$e^{\int P dx} = e^{\int \frac{2x}{x^2+1} dx} = e^{\log(x^2+1)} = x^2+1$$

∴ Soln of D.E is

$$y \cdot e^{\int P dx} = \int e^{\int P dx} \cdot Q \, dx + c$$

$$\therefore y \cdot (x^2+1) = \int (x^2+1) \cdot \frac{1}{(x^2+1)^2} \cdot dx + c$$

$$= \int \frac{1}{x^2+1} dx + c$$

$$y \cdot (x^2+1) = \tan^{-1} x + c$$

ii) Evaluate $\int_0^5 \int_{2-x}^{2+x} dy \, dx + \int_0^2 \int_y^{2y} dx \, dy$

$$= \int_0^5 (y)_{2-x}^{2+x} dx + \int_0^2 (x)_y^{2y} dy$$

$$= \int_0^5 [(2+x)-(2-x)]\,dx + \int_0^2 (2y-y)\,dy$$

$$= \int_0^5 2x\,dx + \int_0^2 y\,dy$$

$$= \left(x^2\right)_0^5 = 25-0 = 25.$$

$$+ \left(\frac{y^2}{2}\right)_0^2 = \frac{4}{2}-0 = 2$$

$\therefore 25+2 = 27$

Ans= 27

Q.2 B) Solve by Method of variation of parameter

$$(D^2+k^2)y = \tan kx$$

→ The A.E is $D^2+k^2=0$

$$\therefore m^2+k^2=0$$

$$m^2 = -k^2 \quad \therefore m = \pm ki$$

$$\therefore C.F = c_1 \cos kx + c_2 \sin kx$$

Assume $P.I = uy_1 + vy_2$

$$y_1 = \cos kx \qquad y_2 = \sin kx$$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos kx & \sin kx \\ -k\sin kx & k\cos kx \end{vmatrix}$$

$$= k\cos^2 kx + k\sin^2 kx$$

$$= k(\cos^2 kx + \sin^2 kx)$$

$$= k(1) = k$$

$$u = -\int \frac{y_2 X}{W}\,dx = -\int \frac{\sin kx}{k} \cdot \tan kx\,dx$$

$$= -\int \frac{\sin kx}{k}\left(\frac{\sin kx}{\cos kx}\right)dx = -\frac{1}{k}\int \frac{\sin^2 kx}{\cos kx}\,dx$$

$$= -\frac{1}{k}\int \frac{1-\cos^2 kx}{\cos kx}\,dx$$

$$= -\frac{1}{k}\int (\sec kx - \cos kx)\,dx$$

$$= -\frac{1}{k}\left(\frac{\log(\sec kx + \tan kx)}{k} - \frac{\sin kx}{k}\right)$$

$$= -\frac{1}{k^2}\log(\sec kx + \tan kx) + \frac{1}{k^2}\sin kx.$$

$$v = \int \frac{y_1 X}{W}\,dx$$

$$= \int \frac{\cos kx}{k}\cdot \tan kx\,dx = \int \frac{\cos kx}{k}\cdot\frac{\sin kx}{\cos kx}\,dx$$

$$= \frac{1}{k}\int \sin kx\,dx = \frac{1}{k}\left(\frac{-\cos kx}{k}\right) = -\frac{1}{k^2}\cos kx$$

$$\therefore P\cdot I = uy_1 + vy_2$$

$$= \left(-\frac{1}{k^2}\log(\sec kx + \tan kx) + \frac{1}{k^2}\sin kx\right)\cos kx$$

$$-\frac{1}{k^2}\cos kx \sin kx$$

$$= -\frac{\cos kx}{k^2}\log(\sec kx + \tan kx)$$

$$\therefore G\cdot S = y = C\cdot F + P\cdot I$$

$$= c_1\cos kx + c_2 \sin kx$$

$$-\frac{\cos kx}{k^2}\log(\sec kx + \tan kx)$$

Q.2c i) Evaluate $\int_0^{\infty} x^2 e^{-x^4} dx$

put $x^4 = t$

$x = t^{1/4}$

$dx = \frac{1}{4} t^{-3/4} dt$

when $x=0, \quad t=0$

$x=\infty, \quad t=\infty$

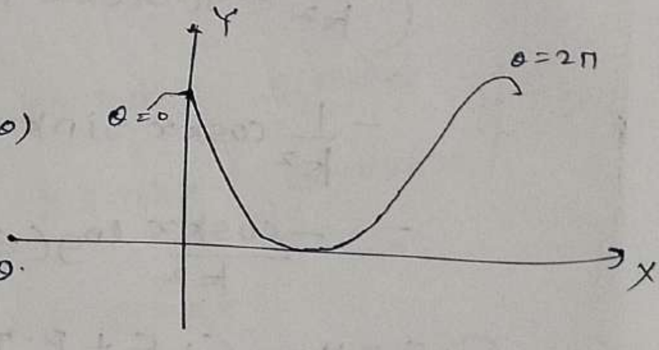
$\therefore I = \int_0^{\infty} t^{1/2} \cdot e^{-t} \cdot \frac{1}{4} t^{-3/4} dt$

$= \frac{1}{4} \int_0^{\infty} e^{-t} \cdot t^{\frac{1}{2}-\frac{3}{4}} dt$

$= \frac{1}{4} \int_0^{\infty} e^{-t} \cdot t^{-1/4} dt$

$= \frac{1}{4} \Gamma 3/4 .$

Q.2 c ii)



$x = a(\theta - \sin\theta), \; y = a(1+\cos\theta)$

$\frac{dx}{d\theta} = a(1-\cos\theta); \; \frac{dy}{d\theta} = -a\sin\theta.$

$\therefore \quad S = \int_0^{2\pi} \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} \cdot d\theta$

$= \int_0^{2\pi} \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta} \cdot d\theta$

$$= \int_0^{2\pi} \sqrt{a^2(1-2\cos\theta+\cos^2\theta)+a^2\sin^2\theta} \cdot d\theta$$

$$= a\int_0^{2\pi} \sqrt{2-2\cos\theta}\, d\theta$$

$$= a\int_0^{2\pi} \sqrt{2.2\sin^2\theta/2}\, d\theta$$

$$= 2a\int_0^{2\pi} \sin\theta/2\, d\theta = 2a\left(\frac{-\cos\theta/2}{1/2}\right)_0^{2\pi}$$

$$= -4a(\cos\pi - \cos 0)$$

$$= -4a(-1-1) = -4a(-2) = 8a.$$

Q.2D Prove that $\int_0^\infty \frac{e^{-ax}\sin x}{x}dx = \cot^{-1}a.$

$\rightarrow$ Let $I(a) = \int_0^\infty \frac{e^{-ax}\sin x}{x}dx$

By D.U.I.S.

$$\frac{dI}{da} = \int_0^\infty \frac{\sin x}{x}e^{-ax}(-x)dx$$

$$= -\int_0^\infty e^{-ax}\sin x\, dx$$

$$= -\left[\frac{e^{-ax}}{a^2+1}(-a\sin x-\cos x)\right]_0^\infty$$

$$= -\frac{1}{a^2+1}\left[0-(0-1)\right] = -\frac{1}{a^2+1}$$

$\therefore \frac{dI}{da} = -\frac{1}{a^2+1}$

$\therefore$

$I(a) = -\tan^{-1} a + c$

put find c, put $a=0$

$\therefore I(0) = -\tan^{-1} 0 + c$

$\therefore \int_0^\infty \frac{\sin x}{x} dx = c$

But $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$ $\therefore c = \pi/2$

$\because \int_0^\infty \frac{e^{-\beta x} \sin \alpha x}{x} dx = \tan^{-1} \alpha/\beta$

put $\beta = 0$, $\alpha = 1$

$\therefore \int_0^\infty \frac{\sin x}{x} dx = \tan^{-1} 1/0 = \pi/2$

$\therefore I(a) = -\tan^{-1} a + \pi/2$

$\therefore I(a) = \cot^{-1} a$

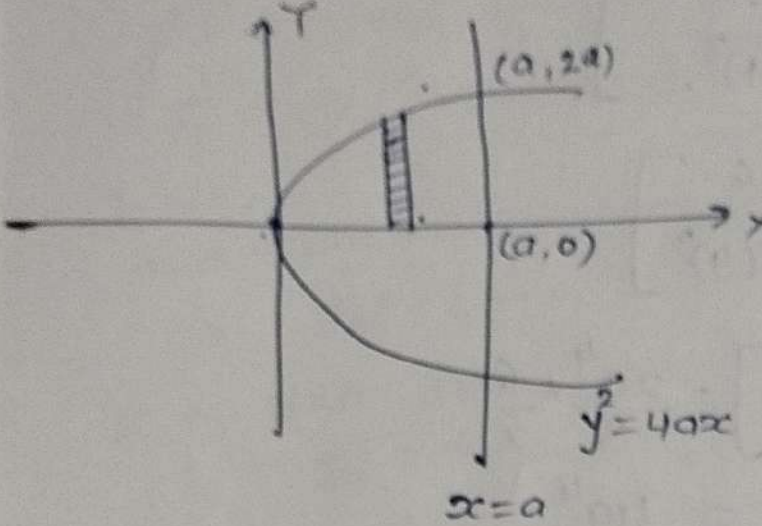
$\therefore \int_0^\infty e^{-ax} \frac{\sin x}{x} dx = \cot^{-1} a.$

Q.2 E) change the order of integration and evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2 \, dy \, dx$

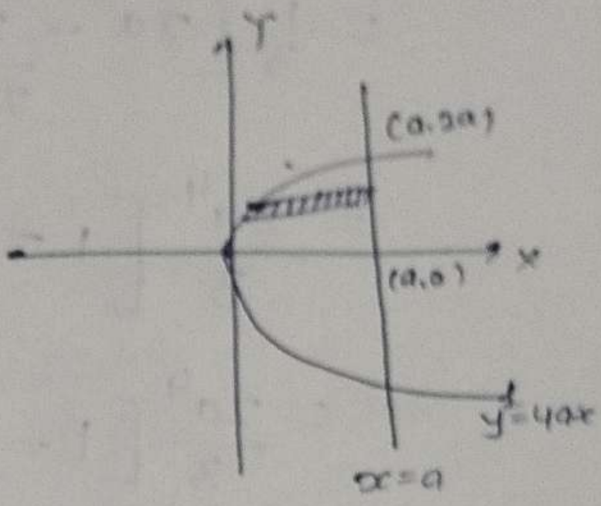
$\rightarrow$ The region of integration is bounded by $y=0$ - X-axis.

$y = 2\sqrt{ax}$ i.e $y^2 = 4ax$ a parabola opening right.

$x = 0$ Y-axis & $x = a$ a line parallel to Y-axis



Before



After

If we change the order of integration

x varies from $x = \frac{y^2}{4a}$ to $x = a$

& y varies from $y = 0$ to $y = 2a$

$$\therefore I = \int_0^a \int_0^{2\sqrt{ax}} x^2 \, dy \, dx = \int_0^{2a} \int_{y^2/4a}^{a} x^2 \, dx \, dy$$

$$= \int_0^{2a} \left(\frac{x^3}{3} \right)_{y^2/4a}^{a} dy$$

$$= \frac{1}{3} \int_0^{2a} \left[a^3 - \frac{y^6}{(4a)^3} \right] dy$$

$$= \frac{1}{3} \left[a^3 y - \frac{y^7}{7(4a)^3} \right]_0^{2a}$$

$$= \frac{1}{3}\left[a^3(2a) - \frac{(2a)^7}{7(4a)^3} - 0 \right]$$

$$= \frac{1}{3}\left[2a^4 - \frac{2^7 \cdot a^4}{7(4)^3} \right]$$

$$= \frac{2a^4}{3}\left[1 - \frac{2^6}{7(4)^3} \right]$$

$$= \frac{2a^4}{3}\left[1 - \frac{1}{7} \right] = \frac{2a^4}{3} \times \frac{6}{7}$$

$$= \frac{4a^4}{7}$$

Q.2 F. $I = \int_0^6 \frac{x^2}{1+x^3}\, dx$

$h = \frac{b-a}{6} = \frac{6-0}{6} = 1$

$\therefore x$	0	1	2	3	4	5	6
y	0	0.5	0.4444	0.3214	0.2462	0.1984	0.1659
ordinates	y_0	y_1	y_2	y_3	y_4	y_5	y_6

i) Trapezoidal Rule

$I = \frac{h}{2}[X + 2R]$

Here $h = 1$, $X = y_0 + y_6 = 0 + 0.1659$ = Extreme

R = sum of Remaining ordinates

$= y_1 + y_2 + y_3 + y_4 + y_5$

$= 0.5 + 0.4444 + 0.3214 + 0.2462 + 0.1984$

$= 1.7104$

$\therefore I = \frac{1}{2}[0.1659 + 2(1.7104)]$

$= 1.7934 = 1.7934$

ii) Simpson's $\frac{1}{3}^{rd}$ Rule

$I = \frac{h}{3}[X + 2E + 4O]$

$h = 1$,

X = Sum of Extreme ordinates

$= y_0 + y_6 = 0 + 0.1659 = 0.1659$

E = Sum of Even ordinates

$= y_2 + y_4 = 0.4444 + 0.2462 = 0.6906$.

O = Sum of ODD ordinates

$= y_1 + y_3 + y_5$

$= 0.5 + 0.3214 + 0.1984 = 1.0198$

$\therefore I = \frac{1}{3}[0.1659 + 2(0.6906) + 4(1.0198)]$

$= 1.8754$

iii) Simpson's $3/8^{th}$ Rule.

$I = \frac{3h}{8}[X + 2T + 3R]$

$h = 1$, X = Sum of extreme ordinates $= 0.1659$

$= y_0 + y_6 = 0 + 0.1659 = 0.1659$

T = Sum of multiple of three ordinates

$= 0.3214 = y_3$

R = Sum of Remaining ordinates

$= y_1 + y_2 + y_4 + y_5$

$= 0.5 + 0.4444 + 0.2462 + 0.1984$

$= 1.3890$

$\therefore I = \frac{3(1)}{8}\left[0.1659 + 2(0.3214) + 3(1.3890)\right]$

$I = 1.8659$

Q.3 a) Solve

$(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$

→ The given D.E is $Mdx + Ndy = 0$ which is homogeneous. where

$M = 3xy^2 - y^3 \quad N = -2x^2y + xy^2$

$\therefore I.F = \frac{1}{Mx + Ny} = \frac{1}{3x^2y^2 - xy^3 - 2x^2y^2 + xy^3}$

$= \frac{1}{x^2y^2}$

Multiply Given D.E by $\frac{1}{x^2y^2}$

$\therefore$ Given D.E becomes

$\left(\frac{3xy^2 - y^3}{x^2y^2}\right)dx - \left(\frac{2x^2y - xy^2}{x^2y^2}\right)dy = 0$

$\therefore \left(\frac{3}{x} - \frac{y}{x^2}\right)dx - \left(\frac{2}{y} - \frac{1}{x}\right)dy = 0$

which is exact D.E

$\therefore$ its soln is

$\int_{y\ \text{const}} M'dx + \int(\text{Terms in } N' \text{ free from } x)\,dy = c$

$\therefore \int\left(\frac{3}{x} - \frac{y}{x^2}\right)dx + \int\frac{2}{y}dy = c$

$\therefore 3\log x + \frac{y}{x} + 2\log y = c$

Q.3b) Solve $(D^2+4)y = x^2$

$\rightarrow$ The A.E is $m^2+4=0$ $m^2=-4$ $\therefore m=\pm 2i$

$\therefore$ C.F $= c_1\cos 2x + c_2 \sin 2x$

$$P.I = \frac{1}{f(D)}X = \frac{1}{D^2+4}\cdot x^2$$

$$= \frac{1}{4\left(1+\frac{D^2}{4}\right)}\cdot x^2$$

$$= \frac{1}{4}\left(1+\frac{D^2}{4}\right)^{-1} x^2$$

$$= \frac{1}{4}\left(1-\frac{D^2}{4}\right)x^2$$

$$= \frac{1}{4}\left(x^2 - \frac{D^2x^2}{4}\right)$$

$$= \frac{1}{4}\left(x^2-\frac{2}{4}\right) = \frac{1}{4}\left(x^2-\frac{1}{2}\right)$$

$\therefore$ G.S = C.F + P.I

$$= c_1\cos 2x + c_2\sin 2x + \frac{1}{4}\left(x^2-\frac{1}{2}\right).$$

Q.3 c) Evaluate $\int_0^1 \frac{x^7}{\sqrt{1-x^2}}dx$

put $x = \sin\theta$ $\therefore dx = \cos\theta\, d\theta$

when $x=0$, $\theta = 0$

$x=1$, $\theta = \pi/2$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^7\theta}{\sqrt{1-\sin^2\theta}}\cdot\cos\theta\, d\theta = \int_0^{\pi/2}\frac{\sin^7\theta}{\cos\theta}\cdot\cos\theta\, d\theta$$

$$= \int_0^{\pi/2}\sin^7\theta\, d\theta$$

$\therefore p=7,\ q=0$

$$= \frac{1}{2} B\left(\frac{7+1}{2}, \frac{0+1}{2}\right)$$

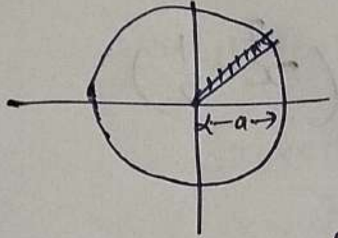
$$= \frac{1}{2} B\left(4, \frac{1}{2}\right) = \frac{1}{2} \frac{\Gamma 4 \, \Gamma 1/2}{\Gamma 9/2}$$

$$= \frac{1}{2} \frac{3 \times 2 \times \Gamma 1/2}{\frac{7}{2} \times \frac{5}{2} \times \frac{3}{2} \times \frac{1}{2} \times \Gamma 1/2}$$

$$= \frac{16}{35}.$$

Q.3 D. change to polar coordinates & evaluate $\iint x \, dx \, dy$ over $x^2+y^2=a^2$

→ put $x = r\cos\theta$, $y = r\sin\theta$, $dx\,dy = r\,dr\,d\theta$



r varies from $r=0$ to $r=a$

$\because x^2+y^2=a^2$

$\Rightarrow r^2=a^2 \Rightarrow r=a.$

& θ varies from $\theta=0$ to $\theta=\pi/2$

$$\therefore I = 4 \int_0^{\pi/2} \int_0^a r\cos\theta \cdot r\,dr\,d\theta$$

$$= 4 \int_0^{\pi/2} \cos\theta \, d\theta \cdot \int_0^a r^2 \, dr$$

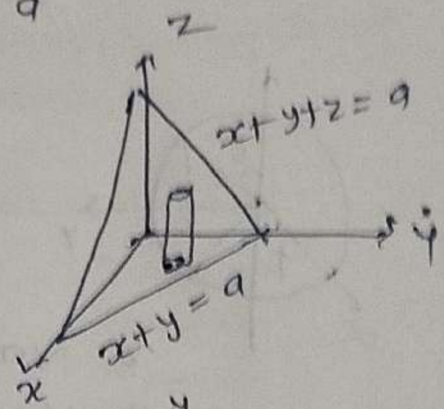
$$= 4 \cdot \frac{1}{2} B\left(\frac{0+1}{2}, \frac{1+1}{2}\right) \cdot \left(\frac{r^3}{3}\right)_0^a$$

$$= 2 B\left(\frac{1}{2}, 1\right) \left(\frac{a^3}{3} - 0\right)$$

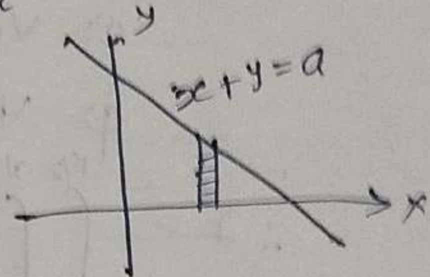
$$= \frac{2a^3}{3} \cdot \frac{\Gamma 1/2 \, \Gamma 1}{\Gamma 3/2} = \frac{2a^3}{3} \cdot \frac{\Gamma 1/2}{\frac{1}{2}\Gamma 1/2} = \frac{4a^3}{3}.$$

Q. 3E. Find the volume of the tetrahedron (17)
bounded by the planes $x=0$, $y=0$, $z=0$ & $x+y+z=a$

$\longrightarrow$ z varies from $z=0$ to $z=a-x-y$
y varies from $y=0$ to $y=a-x$
x varies from $x=0$ to $x=a$



$$\therefore V = \int_{x=0}^{a}\int_{y=0}^{a-x}\int_{z=0}^{a-x-y} dz\,dy\,dx$$



$$= \int_0^a\int_0^{a-x} [z]_0^{a-x-y}\, dy\,dx$$

$$= \int_0^a\int_0^{a-x} (a-x-y)\,dy\,dx$$

$$= \int_0^a \left(ay-xy-\frac{y^2}{2}\right)_0^{a-x} dx$$

$$= \int_0^a \left[a(a-x)-x(a-x)-\frac{(a-x)^2}{2}\right] dx$$

$$= \int_0^a \left(\frac{a^2}{2}-ax+\frac{x^2}{2}\right) dx$$

$$= \left[\frac{a^2}{2}\cdot x-\frac{ax^2}{2}+\frac{x^3}{6}\right]_0^a$$

$$= \frac{a^3}{2}-\frac{a^3}{2}+\frac{a^3}{6}$$

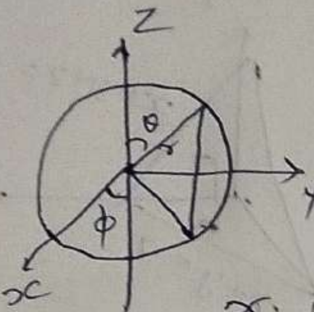
$$= \frac{a^3}{6}.$$

Q. 3 f) Evaluate $\iiint xyz\,dx\,dy\,dz$ over the positive octant of the sphere $x^2+y^2+z^2=a^2$

$\rightarrow$ To transform the integral to spherical coordinates put $x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$.



$dx\,dy\,dz = r^2\sin\theta\,dr\,d\theta\,d\phi$

In the first octant

r varies from $r=0$ to $r=a$

θ varies from $\theta=0$ to $\theta=\pi/2$

ϕ varies from $\phi=0$ to $\phi=\pi/2$

$$\therefore I = \int_0^{\pi/2}\int_0^{\pi/2}\int_0^{a} r\sin\theta\cos\phi\; r\sin\theta\sin\phi\; r\cos\theta\; r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$= \int_0^{\pi/2}\sin^3\theta\,.\cos\theta\,d\theta\int_0^{\pi/2}\sin\phi\cos\phi\,d\phi\,.\int_0^{a} r^5\,dr$$

$$= \left(\frac{\sin^4\theta}{4}\right)_0^{\pi/2}\left(\frac{\sin^2\phi}{2}\right)_0^{\pi/2}\left(\frac{r^6}{6}\right)_0^{a}$$

$$= \left(\frac{1}{4}-0\right)\left(\frac{1}{2}-0\right)\left(\frac{a^6}{6}\right)$$

$$= \frac{1}{4}\times\frac{1}{2}\times\frac{a^6}{6}$$

$$= \frac{a^6}{48}.$$